The Application of Search Theory to Land Search: Adjustment of Probability of Area

by

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Adjustment of Probability of Area (POA)
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Readers are encouraged to review the companion article "Selected Inland Search Definitions" for a detailed description of the terms used herein.[Cooper & Frost, 2000]

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Introduction

A fundamental challenge for those who search for lost people (and objects) is determining where to search. A method adapted from probability theory (a component of Statistics) to help answer this question is initially estimating and then updating or adjusting Probability of Area (POA) values. In its simplest form, POA is defined as the chances that the search area (or region, segment, or any geographic area) contains the object being sought and is usually represented as a percentage (e.g., 45% or 0.45). Although the international SAR community uses a different term than ground searchers (i.e., Probability of Containment or POC), the meaning is the same.[Cooper & Frost, 2000]

Mattson [c1975] introduced a method of establishing initial POA values to the inland search community that required search planners to assess and quantify (reduce to a percentage) their beliefs for each region of probability. These quantified POA values were then averaged across all search planners for each region to produce initial planning POA values. When used correctly, this method ensured that probabilities assigned to the regions were in the correct proportion to one another. Although it was not emphasized at the time, obtaining meaningful POA values requires the use of a proportional assessment technique [Frost, 1999c]. Later modifications to Mattson's [c1975] method did not always capture or preserve proportional relationships. However, the subject of establishing initial POA values is beyond the scope of this article which assumes a valid set of initial POA values has been established.

Once established, POA values may be illustrated for use in the form of a probability map identifying regions of varying probability. As searches are conducted, these initial values are adjusted mathematically to account for changes in probability secondary to the application of detection (POD) in the search area. In the past, adjusting these values and maintaining a probability map required a great deal of computation and time. For search planners, these obstacles were difficult to overcome and, therefore, caused many to disregard the benefits offered by the use of these concepts. In this paper, a mathematical approach to adjusting POA will be advanced that requires less computation and time, thus making the benefits from its use more available for practical application.

Background

The mathematical basis for searching and the study of search theory had its beginnings during World War II in the work of the Navy's Anti-Submarine Warfare Operations Research Group. It was originally based on searching for the wakes of warships as seen from aircraft flying over the ocean. The results of this work were collected in a seminal report by Koopman [1946], but the report was not declassified and generally available until 1958 [Benkoski, et al., 1991]. Koopman [1980] later developed a somewhat expanded version of this work, published in his book *Search and Screening: General Principles with Historical Applications*. Although Koopman's work was aimed at naval interests, the general theory of search he established is applicable to virtually any type of search problem, including that of inland search. [Frost, 1998]

Morse and Kimball [1946] described the use of search probabilities in the maritime environment. However, the inland community did not recognize the concept until Kelly [1973] suggested that a mathematical relationship existed between the probability of success, probability of area, and the probability that "…the victim will be seen…." He, like his maritime counterparts, based this idea on fundamental principles of Bayesian statistics. Bayesian statistics are based on the work of the Reverend Thomas Bayes (1702-1761) who first used probability inductively and established a mathematical basis for probability inference (a means of calculating, from the number of times an event has not occurred, the probability that it will occur in future trials).[ISBA, 1999]

Later, secondary to his research into how the U.S. Coast Guard performed maritime searches, Syrotuck [1975] suggested that a search planner could estimate the chances, or probability, that a particular search would be successful by applying selected mathematical principles. Using a fundamental proposition of probability theory, Syrotuck [1975] described success as a value (defined as the Probability of Success, or POS) calculated as the product of probability of detection (POD) and probability of area (POA) (equation [1]).

[1] $$POS = POD \times POA$$

Where: *POS* is the probability of finding the search object with a particular search.
POD is the probability of the search object being detected, assuming it was in the searched area.
POA is the probability that the search object is contained within the boundaries of the searched area (region, segment, or other geographically defined area).

Syrotuck [1975] succinctly stated that, "Success is based on the capability of 'spotting' the victim and the chances of the victim 'being' in the area that you are searching." Equation [1] illustrates mathematically the concept that the Author articulated: a successful search is dependent on both POD and POA—two related probabilities. Although POS, POD and POA are equally important, this paper will focus on POA and how it changes (or is updated or adjusted) as searches are conducted. However, some fundamental understanding of POS, particularly with regard to its relationship to POA, is necessary to achieve this goal.

As searches are conducted in a segment and nothing is found, the chances that the search object is inside the segment (POA) decrease. This decrease in POA leads to a relative, and directly proportional, increase in the chances of finding the search object—the very definition of POS. The increasing effect of multiple searches on POS is defined as cumulative POS (POScum). As multiple searches are conducted in a segment without results, the POA decreases, and the cumulative POS increases, for that segment. The cumulative POS approaches, but never exceeds, the initial POA of the segment. The POScum values from each of the segments may then be summed to calculate the overall cumulative POS (OPOScum) (equations [2] & [3]) which indicates the overall chances that, if the search object has been in the search area all along, it would have been found by now. [Cooper & Frost, 1999]

[2] $$POScum_s = \sum_{j=1}^{n} POS_{s,j}$$

[3] $$OPOScum = \sum_{s=1}^{m} POScum_s$$

Where:

$POS_{s,j}$ is the probability of success for segment s for search j only. This value does <u>not</u> account for any searching done in segment s before or after search j (j is the search number; i.e., search 1, search 2, … search n).

$POScum_s$ is the cumulative probability of success for segment s only. This value accounts for all searching done to date in segment s. *$POScum_s$* is the sum of all $POS_{s,j}$ values for searches where "j" ranges from 1 to n in segment s. The notation of equation [2] is read as: "Cumulative POS in segment s equals the sum of all $POS_{s,j}$ values where the search number, 'j', goes from 1 to n where n is the number of the most recently completed search of segment s." In short, the cumulative POS for segment s equals the sum of all POS values obtained from all searches conducted to date in segment s.

OPOScum is the overall cumulative probability of success for all searching. This value accounts for all searching done to date in all segments (m is the total number of segments).

Unlike either POD or POA alone, POS is a valid measure of a search's chances of success. That is, POS is a valid measure of search effectiveness. Further, the goal of search planning is to maximize the probability of success. Toward that end, overall cumulative POS can serve as an explicit, computable criteria for comparing alternative search plans and deciding which of them is best.[Soza & Company, 1998].

The benefits of the use of overall cumulative POS as a measure of search effectiveness are available to the search planner only after POA is adjusted for each segment searched. Thus, adjusting POA is an essential step in the use of POS and probability maps as search planning tools. What follows is a brief review of how adjusting POA has been previously described in the inland SAR literature.

Methodological History: Syrotuck

Syrotuck [1975] stated that, “When one area is searched with a certain efficiency [POD] and the victim is not found, this increases the [relative] probability of the victim being in another area.” Put simply, when a particular segment is searched but the search object is not found, two things happen:

1. The relative probability of the object being somewhere else increases, and
2. The relative probability of the object being in the segment just searched decreases.

Mathematically, both of these requirements are satisfied if the POA value of the segment just searched is reduced by an amount proportional to the POD applied to that segment. Syrotuck [1975] called this concept “shifting” POA. Others have called it “adjusting” or “updating” POA. In this context, these terms are synonymous.

When Syrotuck [1975] first attempted to quantify the concept of shifting POA for the inland community, he suggested that POA be adjusted through an application of Bayes’ Theorem in its original form (equation [4]).

[4] $$\text{Shifted POA} = \frac{Pa \times Pm}{Pa \times Pm + Pn}$$

Where: *Shifted POA* is the modified POA of a segment after an unsuccessful search in that segment.
Pa is the probability that the victim is there.
Pm is the probability that the victim was missed.
Pn is the probability that the victim was not there.

The “shifted” POA produced by equation [4] is based on the assumption that only one segment was searched and that all the remaining segment POA values will be “shifted” in the following way: After the “shifted” POA value for a single searched segment is computed, the POA values of the remaining segments are “shifted” so the sum of all “shifted” POAs returns to the original value of 100%. This process is called normalization and the “shifted” POA computed by equation [4] is the correctly normalized value assuming only that segment was searched. In short, Syrotuck [1975] subtracted the “shifted” POA value of the searched segment under consideration from 100% (1.0). The Author then distributed this amount of probability over the remaining segments in proportion to their "unshifted" POA values to get new "shifted" POA values. In the end, the Author's method of normalizing the remaining segments to properly match up with the "shift" in the POA of a searched segment proved to be cumbersome. (see Sidebar 5, page 22, for a description of how to normalize values.)

With Syrotuck's [1975] technique, the entire process just described had to be repeated once for every segment searched in order to produce a "final" set of "shifted" POA values that accounted for all searching done in all searched segments. It is obvious that, in most real situations, this technique involved a very large number of computations. The "shifting" of POAs for even a small number of segments required several pages of calculations. While the premise of the Author's suggestion was valid, its use, and the normalization process specifically, imposed a considerable computational burden on the search planner.

Methodological History: Bownds

About seven years after Syrotuck's [1975] method was published, Bownds [c1982] advanced an alternative, and mathematically equivalent, method for manually adjusting POA values. This method involved a simpler two-step approach to adjusting POA values.

In the first step of Bownds' [c1982] modified approach, the adjusted POA value of the segment just searched (say, segment j) was calculated using equation [5a].

[5a] $$P(Aj \mid Dj') = \frac{(1-Dj) \times Aj}{1-DjAj}$$ {Segment searched}

Where: $P(Aj \mid Dj')$ is the probability that the subject is in segment j (shifted POA) given that segment j was searched without detecting the subject.
Aj is the POA of segment j prior to this update.
Dj is the POD for this search in segment j.

Like Syrotuck's [1975] technique, equation [5a] computes the correctly normalized value assuming only that segment was searched. In Bownds' [c1982] second step, the adjusted, normalized, POA values for each of the remaining segments is calculated by using equation [5b] for all $i \neq j$. As the POA for each searched segment is adjusted, this calculation is performed once for every other segment, including those already adjusted.

[5b] $$P(Ai \mid Dj') = \frac{Ai}{1-DjAj}$$ {Segment not searched}

Where: $P(Ai \mid Dj')$ is the probability that the subject is in segment i ($i \neq j$), given that segment j was searched without results.
Ai is the POA of segment i prior to this update.
Aj is the POA of segment j prior to this update.
Dj is the POD for this search in segment j.

This method ensured the sum of all POA values was equal to 100% at the conclusion of each computational cycle just as Syrotuck's [1975] technique did. In fact, both methods *require* the sum of the POA values be 100% at the end of each computational cycle, just as both methods require as many computational cycles as there are searched segments for a complete update of the POA values.

Although Bownds' [c1982] two-step method appeared considerably simpler than the style of Bayesian update suggested by Syrotuck [1975], it was only a modest improvement in terms of the computational burden imposed on the search planner. Bownds' [c1982] technique also required several pages of calculations for even relatively simple cases. In fact, both authors presented examples of shifting the POA values of a search area having only four segments, all of which had been searched, and each author required two full pages of calculations to obtain the final shifted POA values.

Neither Syrotuck [1975] nor Bownds [c1982] specified whether their terms for POA (i.e., Pa and Ai, respectively) meant shifted POA from previous searches, or whether their terms for POD (i.e., Pm and Di, respectively) represented cumulative POD. If a segment is searched multiple times, the chances of detecting any search object in the segment are increased. This increasing probability of detecting a search object after multiple searches in the same segment is called "Cumulative POD" (equation [6]).

[6] $$\text{Cumulative POD} = 1 - ((1 - POD1) \times (1 - POD2) \times \ldots \times (1 - PODn))$$

Where: *POD1* is the probability of detection of the first search of a segment.
POD2 is the probability of detection of the second search.
PODn is the probability of detection of the nth (last) search.

If both shifted POA from previous searches and cumulative POD were used in the calculations, the new shifted POA values would erroneously account for the search results twice—once in adjusting the POA and once in accumulating POD. Neither Syrotuck's [1975] nor Bownds' [c1982] notation and/or description specifically excluded the possibility of this type of error.

<u>Methodological History: Shea</u>

Shea [1988] published another method for adjusting POA that also required two steps and normalization. However, the Author implied that the calculation of a cumulative POD was required, not optional as in earlier approaches. Further, he implied the use of cumulative POD would allow one to, "…wait until multiple resources have searched an area before calculating the shifted POA." The method allowed the adjustment of POA values after the application of multiple resources. This was a capability not available in previous methods. Indeed, the method could also adjust the POA values for any number of searched segments simultaneously before normalizing. In contrast, the methods of both Syrotuck [1975] and Bownds [c1982] required the adjustment and normalization of the POA value of a single searched segment, followed by the normalization of all other segments, before accounting for additional searched segments in their computations. These older methods took an extraordinary amount of calculation and erroneously implied that normalization was required. In the end, Shea's [1988] method achieved the same mathematical result as Bownds [c1982] and Syrotuck [1975], but required much less computation.

[7a] $$POA^* = (1 - PODcum) \times POAold$$ {Step 1}

Where: *POA** is an interim term needed in step 2 (equation [7b])
POAold is the POA of the segment prior to a search being conducted.
PODcum is the cumulative POD for the segment.

[7b] $$POAshifted = \frac{POA^*}{S}$$ {Step 2}

Where: *POAshifted* is the new POA for each segment.
*POA** is the figure determined from step 1 (equation [7a]).
S is the normalization factor determined from the sum of all POA*.

One thing Shea [1988] did not point out was that the PODcum used in equation [7a] had to be based on only those searches conducted since the last "shifted" POA value was computed for the given segment. If the usual definition of PODcum (cumulative POD for all searching done to date in the segment) is used with a POAold value from an earlier adjustment, the effects of searching done prior to that adjustment will be counted multiple times and errors will result. (Changing POAold to POAinitial, will allow equation [7a] to work correctly with the usual definition of PODcum, as will be shown later. Also, changing PODcum to POD will clarify how the formula works when only one search of the segment(s) has been done since the last POA adjustment.)

Shea [1988] did point out that when using his first equation, the "POA*" value of any segment not searched would be the same as its "POAold." Therefore, performing his step 1 calculation on unsearched segments could be skipped. Further, as justification for his second step (equation [7b]) the Author stated that, "We know that the sum of all POA must be equal to one, so we divide each POA* by S, giving the [normalized] shifted POA." This also implied that normalization was a requirement of the method even though this requirement was based on the false assumption that, "…the sum of all POA must be equal to one," and ignored the possibility of using a defective distribution (a distribution that does not sum to 100%) [Stone, 1989]. The reasons normalization is not required are discussed in "The Normalization Issue" section below.

Shea's [1988] step 1 was a simple algebraic variation of the conventional notation of search probability (POS = POA x POD; equation [1] herein). This was a delightfully simple application of the notation, but the Author, like Syrotuck [1975] and Bownds [c1982], did not completely define his terms. This caused confusion, the potential for miscalculations, and caused doubt about what was otherwise a valid method.

The Normalization Issue

What Syrotuck [1975], Bownds [c1982] and Shea [1988] did not explicitly state was that the process of normalization did not change the relative proportions of the figures derived by removing the denominator from the first step in the first two of these methods and letting Shea's [1988] first step stand unaltered. Normalization may present the figures in a more visually acceptable form (i.e., they add up to 100%), but it is not required if one is only interested in adjusting POA after searching. The additional computational burden of normalizing the figures takes time, is fraught with potential for errors, is unnecessary, and actually destroys valuable information about the search. These are powerful arguments against normalizing.

The U.S. Coast Guard's Computer Assisted Search Planning (CASP) software initially developed in the early seventies uses a very simple algorithm to adjust POA after searches have been conducted. Normalized versions of the non-normalized adjusted POA values are computed within the display modules for presentation purposes using the same normalization technique Shea [1988] used in equation [7b]. However, the CASP software actually uses non-normalized POA values for all POA adjustment calculations and never uses normalized values for anything other than the visual presentation of probability maps. The accuracy of the calculations and the proportionality of the POA values are not affected by the use of non-normalized data [Frost, Pers. Comm., January 1999]. If the inland community used a similar approach, the computational burden of applying search theory to inland searches would be reduced substantially.

Comparative Analysis

From a consideration of the three methods just described the similarity between terms is evident. Table 1 compares these terms (each row is equivalent).

Translation Table			
Syrotuck	Bownds	Shea	Description
Pn	1 - Aj	-	The probability that the subject is not there
Pm	1 – Dj	1 - POD	The probability of not detecting the subject if there
Pa	Aj	POAold	The probability that the subject is there (adjusted for any previous searches)

Table 1 – Term Translation Table

The "cum" was removed from Shea's [1988] rendition to make it consistent, for purposes of comparison, with the other techniques that can handle only one search at a time. Revisiting the formulas for shifting the POA of a searched segment, we see that the numerators of Syrotuck's [1975] and Bownds' [c1982] formulas (equations [4] and [5a]) and the first step of Shea's [1988] method (equation [7a]) are identical. That is,

$$Pa \times Pm = (1 - Dj) \times Aj = (1 - POD) \times POAold$$

{Syrotuck} *{Bownds}* *{Shea}*

In each case, a non-normalized adjusted POA is computed. Shea [1988] showed that normalization could be delayed until after all non-normalized POA values were computed, producing a substantial reduction in the number of calculations required. Below, it will be shown that the number of calculations can be further reduced and that tremendous additional benefits can be realized by eliminating normalization altogether.

In addition to the similarities noted above, the methods of these three authors also share the following characteristics:

1. Because of ambiguous notation, they all have the potential for errors related to accounting for search results twice: once in using adjusting POA from previous searches and once from using cumulative POD.

2. Their concepts and notation do not address the potential need for a second (or more) consensus to be performed with consequent changes in "initial" POA values.

3. All of these methods require normalization which causes the following difficulties:

    a. Normalization requires a great deal more calculation,
    b. Normalization removes the clear indicator (OPOScum) of when to start looking elsewhere.

Difficulties in notation are relatively easy to resolve. However, the importance of the realization that normalization is not required to benefit from the application of search theory cannot be overemphasized. Specifically, using an approach that does not require normalization provides significant benefits including the following:

1. It is computationally simpler with far fewer calculations.

2. It provides a clear indicator (OPOScum) of when the information used to build the scenario that produced the initial POA values (consensus) has been exhausted.

3. Mathematically, the non-normalized figures can be used with or without an initially defective distribution [Cooper, 1998].

4. It abides by the tenets of formal search theory [Frost, 1998].

<u>An Alternative Method</u>

A descriptive and complete notation is required to insure that terms are not confused. The notations illustrated in Sidebar 1 (page 18) will be used to insure accuracy and consistency.

Before continuing, there is an important point that needs clarification so that the same confusion that surrounded Shea's [1988] notation does not resurface. For the purposes of this article, searches of a segment will be accounted for mathematically (e.g., adjusted POA and POS computed) one-at-a-time as their POD values become available. This is also a simple way to handle them in an actual search. Although it is included throughout this discussion, cumulative POD will only be computed just prior to a new consensus, should a new consensus be needed.

Here, cumulative POD will not be used as an input for adjusting POA. This not only relieves the search planner from performing the calculations necessary to compute cumulative POD, it also prevents the errors that can occur when searches are accounted for more than once in adjusting POA. See Sidebar 2 (page 19) for options on how to account for multiple searches of a single segment.

There are two primary reasons one may want to adjust POA values: to update and maintain a probability map to aid in planning future searches, and as part of the process of developing other quantities or values of interest (i.e., OPOScum). Fortunately, algebraic variations of a single equation ([8] in Sidebar 3, page 20) will do both.

Once a search is performed (say in segment s), the only inputs required to compute adjusted POA are:

1. The adjusted POA value that accounted for all searches prior to this search in segment s ($POA_{s,n-1}$ or $POA_{s,0}$ if this is the first search), and

2. The POD for this search of segment s ($POD_{s,n}$).

The primary equations to perform these computations are included in Sidebar 3 (page 20).

As stated earlier, algebraic variations of equation [8], such as those listed in Sidebar 4 (page 21), will also work to calculate adjusted POA. However, the use of the primary equations listed in Sidebar 3 (page 20) will minimize the quantity of computations required. For instance, although equation [15] (in Sidebar 4, page 21) achieves the same result as equation [8], it requires one additional computation (calculating POScum) and therefore is less desirable.

After computing adjusted POA, the search planner will want to update any probability map in use and compute overall cumulative POS. However, given the updated POA, any value of interest could be computed from the algebraic variations of these equations. For example, segment POS for one search could be calculated with equation [16], cumulative segment POS (for all searches to date in this segment) could be calculated with equation [9], and cumulative POD for this segment (for all searches to date in this segment) could be calculated with equation [12].

Algebra aside, no search planner wants to perform unnecessary calculations, so the remainder of this discussion will be limited to the steps necessary to:

1. Update a probability map (adjust POA after a search), and
2. Compute overall cumulative POS.

A method for computing cumulative POD will also be demonstrated because it may be useful if a second (or subsequent) consensus is performed and a new set of "initial" POA values is established for each segment.

An Example

Consider a search area that has been divided into four segments (Segments 1, 2, 3 & 4) as illustrated in Figure 1. In this example, each segment is also a probability region (see the companion article "Selected Inland Search Definitions" for the definitions of "segment" versus "region"). It will be assumed that search planners established regional boundaries based only on factors that affect POA. After a proportion-based consensus was conducted, initial planning POA values ($POA_{s,0}$) were assigned to regions, and segment POA values were derived from these regional values. In this example, the initial POA values of segments 1 through 4 will be 19.35%, 25.81%, 29.03% and 25.81%, respectively (Figure 1).

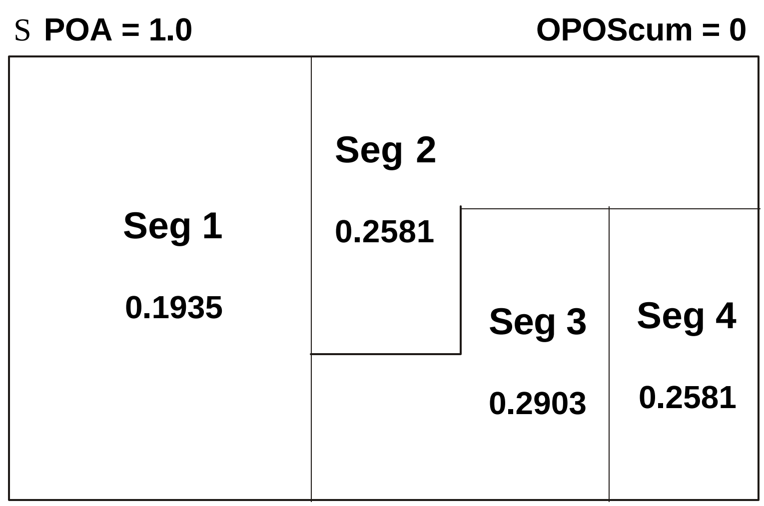

Figure 1 – Probability map with four segments, each showing their respective initial POA values. Note that, since no searching has occurred, the overall cumulative POS is 0.

Search planners have determined that it is possible and appropriate to search all of the segments simultaneously to a Coverage of 0.5 with the available resources. So, search resources are deployed to their respective segments and begin operations.

After a search is conducted and a POD value is established for that search, one would expect the initial POA value for the segment searched to change in some proportion to the POD achieved. In our example, if segment 4 is searched to a Coverage of 0.5, and thus a POD of 39.3%, a number of useful values could be computed. Using equation [8], $POA_{s,p}$ is easily calculated to be 15.67% {0.1567 = 0.2581 × (1 - 0.393)}, and using equation [16], $POS_{s,n}$ is calculated to be 10.14% {0.1014 = 0.2581 – 0.1567}.

The order in which searches are accounted for in the calculations has no effect on the result. But, the POA values are most easily adjusted one-at-a-time. Once a search has been completed and a Coverage assessed using the information available from debriefing, the corresponding POD can be used to adjust the POA value and then the overall cumulative POS can be computed. In the example presented, debriefing information from the search of segment 4 has been presented to search planners first. So, the calculations involving the adjustment of the POA value in segment 4 will be performed first (Table 2).

Search 1

Segment	$POA_{s,0}$	(C=0.5) $POD_{s,n}$	$POA_{s,n}$	$POScum_s$	
1	0.1935	0	0.1935	0	
2	0.2581	0	0.2581	0	
3	0.2903	0	0.2903	0	
4	0.2581	0.393	0.1567	0.1014	
Totals	1.0000		0.899*	0.1014	< OPOScum

Table 2 – Search 1. (*) The adjusted values are not normalized.
That is, the sum of all POA values no longer equals 100% after searching.

Note that when not normalized, the POA values of un-searched segments (e.g., segments 1, 2, and 3 in this example) do not change. The cumulative POS for each segment searched is calculated using equation [9]. The overall cumulative POS is simply the sum of all segment cumulative POS values [Soza, 1998] (see bottom right cell of Table 2) and is mathematically depicted in equation [3]. Also note there is another way to compute OPOScum: the difference between the total of the $POA_{s,0}$ values (1.0 in this example) and the total of the $POA_{s,n}$ values. This method is mathematically depicted in equation [10]. When the probability map is updated with the search described in Table 2, it looks like Figure 2.

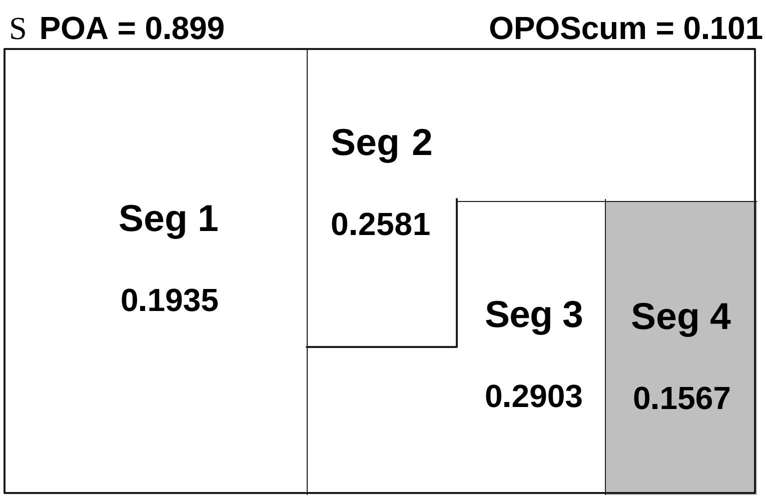

Figure 2 – Probability map showing the adjusted POA value in segment 4 after one segment is searched.

Looking at the updated probability map, search planners can see that segment 4 was searched once and that the probability in that segment dropped from 25.8% (initial POA) to 15.7%. Any other values of interest to the search planner can be calculated from here.

If debriefing information from the search of segment 2 were presented to search planners next (Coverage = 0.5; POD = 39.3%), the values on the probability map would be correspondingly updated (Figure 3).

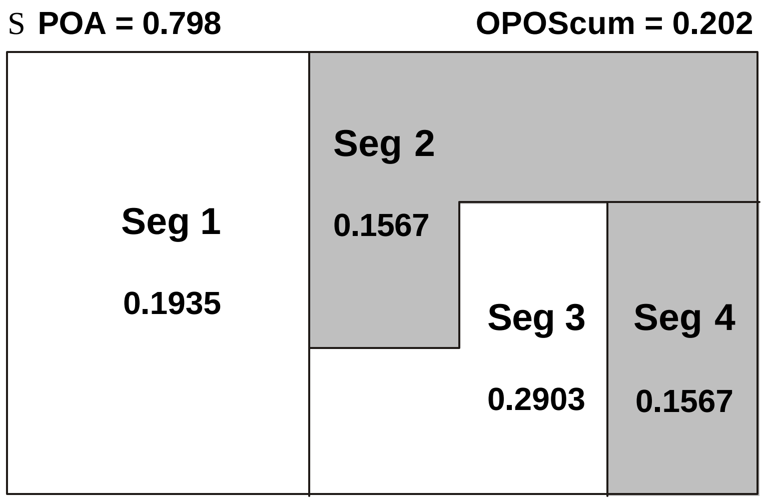

Figure 3 – Probability map showing adjusted POA values after two segments are searched.

This process continues until all searches (and their associated POD values) have been processed and reflected on the probability map. Note that the sum of all POA values at any time, when added to the corresponding OPOScum value at that time, always equals the sum of the initial consensus POA values (1.0 in this example). This is a consequence of equation [10].

<u>Re-adjusting POA Values after a New Consensus</u>

Occasionally, a situation arises where new information about a search is discovered invalidating the scenario originally used to establish initial POA values (via consensus). In these circumstances, a new consensus may be performed. If a new consensus is necessary, and new initial POA values are established, there is no need to discard all searching (e.g., Coverage or POD) previously conducted. All previous searching may be accounted for using any one of three mutually exclusive methods:

1. Mathematically
2. Subjectively
3. By excluding regions/segments previously searched from consideration in the new consensus

If the search area is unchanged, or at least some of the segments previously searched remain in the new search area, searches conducted prior to the new consensus can be used to:

1. Adjust POA values established in the new consensus to reflect searching already done, and

2. Update the overall cumulative POS to reflect both the new consensus and previous searching.

This is an under-emphasized benefit of the application of search theory using non-normalized values. The ability to re-adjust POA and POS values easily to properly account for both a new consensus and the searching that has already been completed is a powerful capability.

Mathematically. By adding a third subscript, representing the consensus number (i.e., consensus 1, consensus 2, etc.), to terms in equations [12], [11] and [10], we can account for a second consensus.

[12] $$PODcum_s = 1 - \left(\frac{POA_{s,n,1}}{POA_{s,0,1}}\right)$$

[11] $$POA_{s,n,2} = POA_{s,0,2} \times (1 - PODcum_s)$$

[10] $$OPOScum = \Sigma POA_{s,0,2} - \Sigma POA_{s,n,2}$$

To mathematically re-adjust POA and POS values after a second consensus, these equations may be used in the following way:

1. Equation [12] (or [6], but [12] is preferred) is used to compute the cumulative POD ($PODcum_s$) for each segment in the search area prior to the new consensus.

2. These cumulative POD values are entered in equation [11], with the initial POA values established by the second consensus ($POA_{s,0,2}$), to get new adjusted POA values ($POA_{s,n,2}$) for each segment.

3. Once all the re-adjustments have been made for the second consensus (steps 1 and 2, above), the overall cumulative POS ($OPOScum$) may be computed using equation [10].

Subjectively. Rather than mathematically accounting for previous searching after a new consensus, a similar goal can be achieved by subjectively considering any searching done to date *during* the new consensus. This alternative involves search planners subjectively evaluating all previous searching done in the search area as they would anything that might influence their assessment during the new consensus process (e.g., evidence). If this approach is taken, all previous POD or cumulative POD values are discarded and the new POA values established from the new consensus are considered initial POA values for the search (e.g., $POA_{s,0}$). As expected, cumulative POS values (both segment and overall) also return to zero.

Excluding regions/segments previously searched. A situation may arise where new information about a search is discovered invalidating the scenario originally used to establish the initial search area. In this situation, there are four primary options:

1. The entire search area may be moved, leaving no segments in common with the previous search area,

2. Segments may be added to the previous search area without discarding any part of the previous search area,
3. Part of the previous search area may be discarded because it no longer fits the scenario in use, or
4. Part of the previous search area is discarded AND segments are added (a combination of 2 and 3).

For options 1, 2 and 4, any segments added to the new search area have, of course, not been previously searched, so accounting for previous searches in them is unnecessary. For options 3 and 4, any previous searching that has been done in the parts of the search area that have been discarded will no longer have any impact on future search planning. Thus, if the parts of the search area that have been previously searched are excluded from consideration in the new consensus, the need to account for previous searching in these areas is unnecessary. On the other hand, any segments previously searched that are included in the search area developed by the second consensus must have their POA values properly adjusted by either a mathematical or subjective procedure. Then, further adjustments accounting for still more searching after the second consensus may proceed using equation [8] and the approach previously described. The same procedures would also apply should a third (or more) consensus be required.

With this powerful capability to account for previous searching after a new consensus comes a word of caution. If planners account for previous searching in multiple ways (i.e., both mathematically and subjectively), the resulting planning POA values will not accurately represent the current situation, the consensus of planners or all the information available. Therefore, caution must be exercised to ensure that any relevant previous searching is accounted for with only one method after a new consensus is performed.

Summary

It has been shown that many values of interest to search planners, such as overall search effectiveness to date (OPOScum), the relative (and absolute) POA values of the segments, and even segment cumulative POD values, can be made available simply by tracking the non-normalized adjusted POA values. Although a full description of their use is beyond the scope of this document, when used properly, OPOScum and the shrinking non-normalized POA values provide clear indicators of when it may be time to consider other scenarios, or at least re-evaluate the information used to create the current one. It has also been shown that tracking the non-normalized POA values preserves enough information about the searching done to date to permit proper readjustment to account for a new consensus that changes the initial POA values. Finally, in the quest for simplicity, a primary benefit of this practical application of search theory has been demonstrated: it requires substantially fewer computations than previously suggested.

Exercises for the Reader (Answers in Sidebar 6, page 23)

1. Suppose search conditions in segments 1 and 3 (see Figure 2) were better than expected, making the Coverage 0.7 (POD = 50%) in segment 1 and 0.6 (POD = 45%) in segment 3. Compute the new adjusted POA and OPOScum values and create a new probability map reflecting these results.

2. Suppose all segments are searched a second time to a Coverage of 0.36 (POD = 0.30). Adjust all the POA values and compute the OPOScum.

3. Now, suppose new information comes to light and a new consensus is performed resulting in the following new initial segment POA values: 1 – 10%, 2 – 40%, 3 – 20%, 4 – 30%. What are the new adjusted POA values based on the new consensus and the two searches of each segment that have already been completed? What is the new OPOScum value?

<u>References</u>

Benkoski, M., Monticino, M., & Weisinger, J. (1991). "A Survey of the Search Theory Literature." *Naval Research Logistics, 38*, 469-494.

Bownds, J.M. (c1982). *A Simple Version of the Bayesian Information Processing for Updated POA Estimates*. Private Publication, Desert SAR Group, Department of Mathematics, University of Arizona.

Bownds, J.M., Ebersole, M., Lovelock, D. and O'Connor, D. (1990). *Searchbusters: Reexamining the Search Management Function*. Private publication, Phoenix, Arizona: Authors.

Bownds, J.M., Ebersole, M., and Lovelock, D. (1992). *Computer Aided Search Information Exchange III (C.A.S.I.E. III)*. Software application, Phoenix, Arizona: Authors.

Cooper, D.C., and Frost, J.R. (2000). *Selected Inland Search Definitions*. Private publication, Cuyahoga Falls, Ohio: Authors.

Cooper, D.C. (1998). "An Outline of Selected Concepts of Land Search Theory." *NASAR Conference Proceedings*, Response '98, 27th Annual Conference and Exhibition of the National Association for Search and Rescue, 27-30 May 1998, pp 83-106.

Frost, J.R. (1998). *Search Theory Enhancement Study*. Prepared for Interagency Committee on Search and Rescue Research and Development Working Group. Fairfax, Virginia: Soza and Company, Ltd.

Frost, J.R. (1999a). "Principles of Search Theory, Part I: Detection." *Response, 17(2)*, pp. 1-7.

Frost, J.R. (1999b). "Principles of Search Theory, Part II: Effort, Coverage, and POD." *Response, 17(2)*, pp. 8-15.

Frost, J.R. (1999c). "Principles of Search Theory, Part III: Probability Density Distributions." *Response, 17(3)*, pp. 1-10.

Frost, J.R. (1999d). "Principles of Search Theory, Part IV: Optimal Effort Allocation." *Response, 17(3)*, pp. 11-23.

IMO/ICAO. (1999a). *International Aeronautical and Maritime Search and Rescue Manual*: *Vol. I. Organization and Management*. London/Montreal: the International Maritime Organization (IMO) and the International Civil Aviation Organization (ICAO).

IMO/ICAO. (1999b). *International Aeronautical and Maritime Search and Rescue Manual*: *Vol. II. Mission Co-ordination*. London/Montreal: the International Maritime Organization (IMO) and the International Civil Aviation Organization (ICAO).

IMO/ICAO. (1999c). *International Aeronautical and Maritime Search and Rescue Manual*: *Vol. III. Mobile Facilities*. London/Montreal: the International Maritime Organization (IMO) and the International Civil Aviation Organization (ICAO).

International Society for Bayesian Analysis (ISBA) (1999). *Who was the Reverend Thomas Bayes?* WEB: http://www.bayesian.org/bayesian/bayes.html.

Kelly, D. (1973). *Mountain Search for the Lost Victim*. Montrose, California: Author.

Koopman, B.O. (1946). *Search and Screening: OEG Report 56.* Operations Evaluations Group, Office of the Chief of Naval Operations, U.S. Navy Department, Washington D.C.

Koopman, B.O. (1980). *Search and Screening: General Principles with Historical Applications.* New York, NY, USA: Pergamon.

Mattson, R.J. (c1975). *Establishing Search Priorities*. Private Publication.

Morse, P.M. and Kimball, G.E. (1946). *Methods of Operations Research: OEG Report 54.* Operations Evaluations Group, Office of the Chief of Naval Operations, U.S. Navy Department, Washington D.C.

Shea, G. (1988). "Formula for the Field: A Simple Method for Shifting Probability of Area." *Response.* Vol. 7, No. 3, pp. 20-24.

Soza and Company, Ltd. & U.S. Coast Guard. (1998). *The Theory of Search: a Simplified Explanation.* (Rev. ed.). Fairfax, Virginia: Authors.

StatSoft, Inc. (1999). *Electronic Statistics Textbook.* Tulsa, OK: StatSoft. WEB: http://www.statsoftinc.com/textbook/stathome.html.

Stone, L.D. (1989). *Theory of Optimal Search*, Second Edition, Arlington, Virginia: Operations Research Society of America.

Syrotuck, W.G. (1975). *An Introduction to Land Search Probabilities and Calculations.* Arner Publications, Westmoreland, New York, 13490, USA.

Sidebars

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Sidebar 1 – Notation Description

The first subscript (s) designates the segment or region, i.e., segment s, in this example.

The second subscript (n) designates the search number, i.e., search number n, in this example.

This is the adjusted POA value of segment s after search n, and is based on consensus c.

This is the adjusted POA value of segment s prior to search k, and is based on consensus c.

"k-1" designates the search just prior to search k.

$$POA_{s,n,c} = POA_{s,k-1,c} \times (1 - PODcum_{s,(k,\ldots n)})$$

The third subscript (c) designates the consensus number; that is, consensus 1, consensus 2, etc. This subscript will usually be omitted and presumed to be 1 (the first consensus) unless otherwise specified.

This is the cumulative POD for segment s where (k,…n) is optional and denotes the searches (k through n) included in the computation. For example, $PODcum_{s,(3,4,5)}$ denotes the cumulative POD for searches 3, 4, and 5 only in segment s.

If the (k,…n) subscript is not shown with $PODcum_s$, the inclusion of all searches (i.e., 1, … n) in segment s is implied, making k = 1 everywhere in this equation.

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Sidebar 2 – Accounting for Multiple Searches of a Segment

If one desires to account for multiple searches (and thus multiple POD values), **one** of the following approaches must be used. To illustrate the point, five searches (searches 1, 2, 3, 4 and 5) will be conducted in segment A1. In each example below, searches 1 and 2 have been completed and accounted for in the adjusted POA. The examples show alternative methods for computing an adjusted POA that accounts for all five searches. Note that all of the following equations are algebraic variations of equation [8].

1. Adjust POA after each individual search by dealing with each POD value one-at-a-time until all have been accounted for. The following minor variation of equation [8] is computed after each search.

   [8'] $$POA_{\mathbf{A1,n}} = POA_{\mathbf{A1,n-1}} \times (1 - POD_{\mathbf{A1,n}})$$

   Where:

   n goes from 3 to 5 to account for the third, fourth and fifth searches in order.
   $POA_{A1,n}$ is the adjusted POA value in segment A1 for search n only.
   $POA_{A1,n-1}$ is the adjusted POA value in segment A1 (based on the initial consensus POA value established in segment A1) for the search just before n (e.g., n minus1).
   $POD_{A1,n}$ is the POD applied in segment A1 for search n only.

2. Compute cumulative POD ($PODcum_{A1,(3,4,5)}$) for the searches 3, 4 and 5 using equation [6] and then multiply $POA_{A1,2}$ (the adjusted POA just prior to the first of the multiple searches—after accounting for search 2) by this value to get the new adjusted POA ($POA_{A1,5}$). The following variation of equation [8] is computed after searches 3, 4 and 5 have been completed.

   [8"] $$POA_{\mathbf{A1,5}} = POA_{\mathbf{A1,2}} \times (1 - PODcum_{\mathbf{A1,(3,4,5)}})$$

   Where:

   $POA_{A1,5}$ is the adjusted POA value in segment A1 for all 5 searches.
   $POA_{A1,2}$ is the adjusted POA value in segment A1 for searches 1 and 2.
   $PODcum_{A1,(3,4,5)}$ is the cumulative POD accounting for searches 3, 4 and 5 only in segment A1.

3. Compute the cumulative POD ($PODcum_s$) for all 5 searches in segment A1 using equation [6] and multiply the initial (consensus) $POA_{s,0}$ by this value to get the current adjusted POA.

   [8'''] $$POA_{\mathbf{A1,5}} = POA_{\mathbf{A1,0}} \times (1 - PODcum_{\mathbf{A1,(1,2,3,4,5)}})$$

   Where:

   $POA_{A1,5}$ is the adjusted POA value in segment A1 for all 5 searches.
   $POA_{A1,0}$ is the initial (consensus) POA value in segment A1 prior to any searching.
   $PODcum_{A1,(1,2,3,4,5)}$ is the cumulative POD accounting for all 5 searches in segment A1 and is equivalent to $PODcum_{A1}$, the conventional definition of cumulative POD.

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Sidebar 3 – Primary Equations

[8] $$POA_{s,n} = POA_{s,n-1} \times (1 - POD_{s,n})$$

[9] $$POScum_s = POA_{s,0} - POA_{s,n}$$

[10] $$OPOScum = \Sigma POA_{s,0} - \Sigma POA_{s,n}$$

[3] $$OPOScum = \sum_{s=1}^{m} POScum_s$$

Where:

$POA_{s,n}$ is the adjusted POA value in segment s (based on the initial POA value established in consensus c for region or segment s) after all searches (hereafter, c shall be presumed to be 1 unless otherwise specified). This value accounts for all searching done to date in segment s.

$POA_{s,n-1}$ is the adjusted POA value in segment s (based on the initial POA value established in consensus c for segment s) for the specific search just before n (e.g., n minus1). This value accounts for all searching done in segment s prior to search n.

$POD_{s,n}$ is the probability of detection for search n in segment s. This value is <u>not</u> a cumulative value and indicates the POD for search n only.

$POScum_s$ is the cumulative POS for segment s only. This value accounts for all searching done in segment s.

OPOScum is the overall cumulative POS. This value accounts for all searching done in all regions and/or segments.

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Sidebar 4 – Additional Useful Equations (Algebraic Equivalents)

[11] $$POA_{s,n} = POA_{s,0} \times (1 - PODcum_s)$$

[12] $$PODcum_s = 1 - \left(\frac{POA_{s,n,c}}{POA_{s,0,c}}\right)$$

[13] $$POA_{s,n} = POA_{s,n-1} - POS_{s,n}$$

[14] $$POA_{s,n} = POA_{s,n-1} - (POD_{s,n} \times POA_{s,n-1})$$

[15] $$POA_{s,n} = POA_{s,0} - POScum_s$$

[16] $$POS_{s,n} = POA_{s,n-1} - POA_{s,n}$$

[17] $$POScum_s = POA_{s,0} \times PODcum_s$$

[18] $$PODcum_s = \frac{POScum_s}{POA_{s,0}}$$

Where:

$POA_{s,n}$ is the adjusted POA value in segment s (based on the initial POA value established in consensus c for region or segment s) after all searches (hereafter, c shall be presumed to be 1 unless otherwise specified). This value accounts for all searching done in segment s up to and including search n.

$POA_{s,n-1}$ is the adjusted POA value in segment s (based on the initial POA value established in consensus c for segment s) for the specific search just before n (i.e.., n minus1). This value accounts for all searching done in segment s prior to search n.

$POD_{s,n}$ is the probability of detection for search n in segment s. This value is not a cumulative value and indicates the POD for search n only.

$POS_{s,n}$ is the POS for segment s for search n only. This value does not account for any searching done in segment s before or after search n.

$POScum_s$ is the cumulative POS for segment s only. This value accounts for all searching done in segment s.

OPOScum is the overall cumulative POS. This value accounts for all searching done in all regions and/or segments.

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Sidebar 5 – Normalizing Values

Any time normalized POA values are desired for presentation purposes, the "normalization factor" (S in Shea's [1988] notation), is computed as follows:

1. Compute the sum of all non-normalized adjusted POA values (Σ POA).

2. Divide this value by the sum of all the initial POA values (Σ $POA_{s,0}$; prior to any searching or adjusting) to get the "normalization factor," S. This step accounts for initially defective distributions (when the sum of all initial POA values does not equal 100%). It is not needed if the initial distribution was complete.

3. Then, divide all the non-normalized POA values by the "normalization factor" to get the normalized POA values.

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Sidebar 6 – Exercise Answers

Exercise 1. Computing the new adjusted POA values for two more searches is done in the same way that the first two searches were done. Figure 4 illustrates the use of equations [8] and [10] and the resulting POA values after adjusting for segment 1 being searched to a Coverage of 0.7 (POD = 0.50), and Figure 5 shows the results of segment 3 being searched to a Coverage of 0.6 (POD = 0.45).

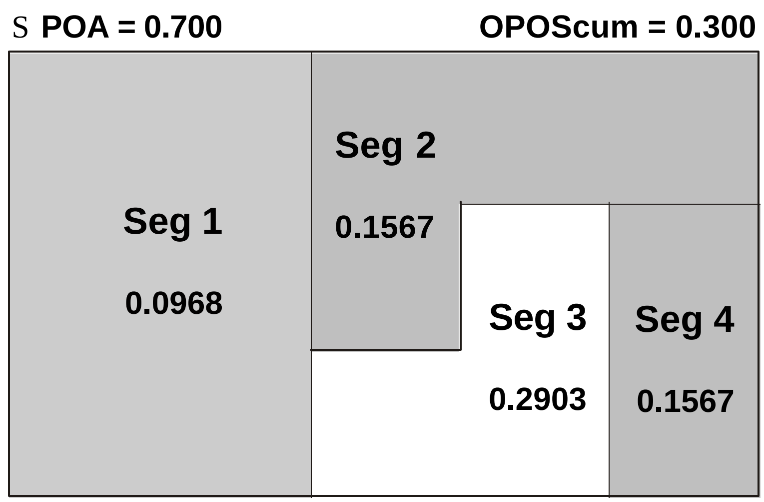

Figure 4 – Probability map showing adjusted POA values after three segments searched.

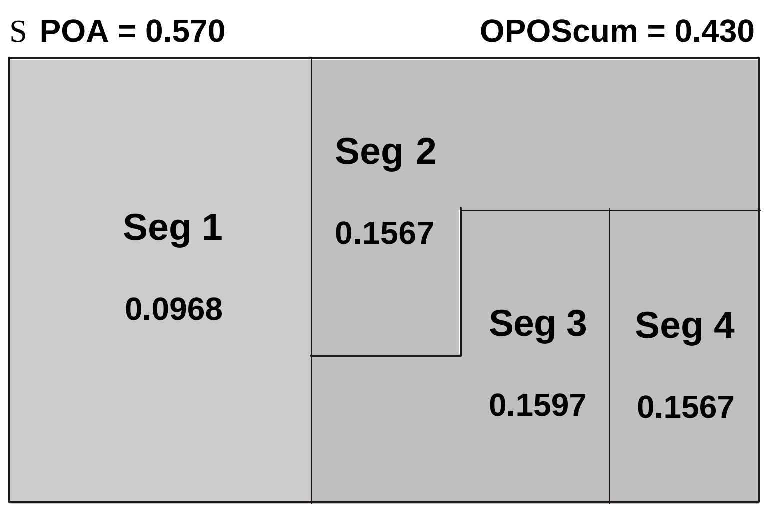

Figure 5 – Probability map showing adjusted POA values after all four segments searched.

Exercise 2. To determine the new adjusted POA values for each segment following the second search of each one, use equation [8] to apply the POD of 30% (0.3) to each previous adjusted POA. Then use equation [10] to compute the value of OPOScum. Table 3 summarizes the results.

Segment	$POA_{s,0}$	$POD_{s,1}$	$POA_{s,1}$	$POD_{s,2}$	$POA_{s,2}$	OPOScum
1	0.1935	0.500	0.0968	0.300	0.0678	
2	0.2581	0.393	0.1567	0.300	0.1097	
3	0.2903	0.450	0.1597	0.300	0.1118	
4	0.2581	0.393	0.1567	0.300	0.1097	
Totals	1.0000		0.5699		0.3990	0.6011

Table 3 – Summary of results for Exercise 2.

Exercise 3. To determine the new adjusted POA values for each segment after the second round of searches and a subsequent new consensus, the $PODcum_s$ for each segment must first be computed. Either equation [6] or [12] will work for this. However, equation [12] requires just one division and a subtraction no matter how many searches of the segment have been done and it does not require knowing the POD of any completed search. (All of those POD values have already been accounted for in the POA adjustments, if done correctly.) Equation [6], on the other hand, adds a term for each completed search and requires the search planner to look up the POD of every completed search as input. Thus, equation [12] is the preferred method. Once the $PODcum_s$ has been computed for each segment, equation [11] can be used to compute the adjusted POA value which accounts for both the new consensus and all previous searches ($POA_{s,n,2}$). The overall cumulative POS (OPOScum) can then be computed using equation [10]. Table 4 summarizes these results and Figure 6 illustrates a probability map that accounts for all previous searching as well as the new consensus.

Segment	$POD_{s,1}$	$POD_{s,2}$	$PODcum_s$	$POA_{s,0,2}$	$POA_{s,1,2}$	OPOScum
1	0.500	0.300	0.650	0.100	0.035	
2	0.393	0.300	0.575	0.400	0.170	
3	0.450	0.300	0.615	0.200	0.077	
4	0.393	0.300	0.575	0.300	0.128	
Totals				1.000	0.410	0.590

Table 4 – Results of two series of searches and a second consensus.

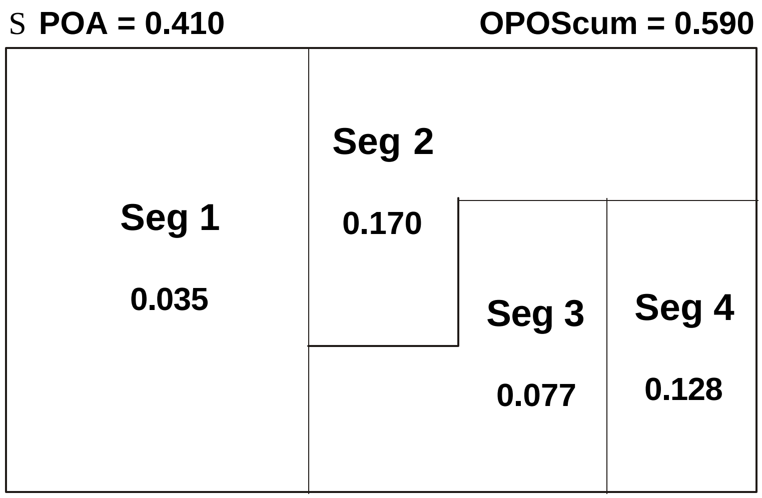

Figure 6 – Probability map showing adjusted POA values after second consensus and accounting for all previous searching.